# Programming and Proving with Indexed Effects

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Proving properties about effectful programs is hard. New application-specific abstractions based on indexed monads can help simplify programming and proving. However, existing languages lack support to develop and use such abstractions.

The main contribution of this paper is a type-and-effect system that enables program proof developers to design new effect-typing disciplines based on indexed monads, making proofs simpler and more abstract and allowing programs to be developed in a direct, applicative syntax while automatically elaborating them into a core language of pure, total functions where the monadic structure is made explicit.

We have implemented our system as a new feature in the F$^\star$ programming language, enhancing its existing user-defined effect system to cover *all* forms of indexed monads. In doing so, we have also simplified the core language of F$^\star$, allowing us to derive basic Dijkstra monad constructions in F$^\star$ that were previously primitive.

Finally, we present several case studies developing new indexed monad constructions to structure program proofs in settings including information flow control, algebraic effects, and low-level binary format parsers.

## 1 INTRODUCTION

Xavier Leroy, in his Royal Society Milner Award lecture,[1] claims that purely functional programming is the shortest path to writing and proving a program. Few practitioners of program proofs will disagree—programming and reasoning in the presence of computational effects is hard. However, effectful programming is indispensable in many domains, e.g., when building low-level or high-performance software. We are interested in techniques that simplify the construction of proofs of correctness and security of effectful programs, starting by representing effects using *monads*.

While Moggi [1989] and Wadler [1992] firmly established monads as both a categorical and programmatic basis on which to develop effectful programs, proving the correctness of programs with monadic effects is somewhat less settled. Several researchers have proposed variants of monads, with richer indexing structure, in support of more precise static reasoning. Many of these proposals have been profitably used in a variety of settings, yet no single proposal has emerged as universal—given the diversity of program reasoning tasks, universality of structures in support of reasoning is hard to expect or imagine. Rather, we embrace the diversity of indexed monads and seek to use them to structure and simplify program proofs. We briefly survey the landscape.

*Monads.* From a programmer's perspective, a monad `M` is a typeclass representing an effectful computation, supporting the following two combinators: `return: a → M a`, to promote a pure value to an `M` computation; and `bind: M a → (a → M b) → M b`, to sequentially compose two `M`-computations, where `bind` is associative and `return` is both a left and right unit of `bind`. A great many computational effects have been shown to be expressible as monads, including state, exceptions, continuations, parsing, printing, asynchrony, and many others besides. However, the type `M a` is relatively uninformative. A program with type `M a` may exhibit any of the effects encoded in `M` when executed, e.g., it may read and write the state arbitrarily. When trying to prove a program correct, this imprecision is usually unacceptable, leading to several indexed monad structures with adaptations of the monad laws to account for the indices.

[1] https://xavierleroy.org/talks/Milner-award-lecture.pdf

*Graded monads.* Katsumata's (2014) graded monads are a monad-like typeclass `G` indexed by a monoid $\{\mathtt{I}, \oplus, \epsilon\}$, whose `return` and `bind` have the following signatures: `return : a → G a` $\epsilon$ and `bind : G a i → (a → G b j) → G b (i ⊕ j)`. By choosing the indexing monoid carefully, one can recover some precision in static reasoning. For instance, the indices can be used to constrain the memory locations a computation may write, ensuring, depending on the index `i`, that a given `G a i` computation leaves certain parts of memory unchanged.

*Parameterized monads.* Atkey's (2009) parameterized monads are a typeclass `A` with two indices, with the following combinators: `return : a → A a p p` and `bind: A a p q → (a → A b q r) → A b p r`. Given an `e : A a p q`, the index `p` is an abstraction of the resources expected by `e`, and `q` abstracts the resources remaining after `e` executes. Parameterized monads have been used to reason about a variety of effects, including, for example, message passing programs using session types.

*Hoare monads.* Working in a dependently typed setting, and aiming initially to prove stateful programs correct in Coq, Nanevski et al. [2008] developed the Hoare monad, a typeclass `H`, indexed by memory predicates `p : mem → prop` and `q:a → mem → prop`, with combinators of the form `return : x:a → H a (p x) p`, meaning that a pure computation returns an `x:a` while preserving any predicate `p x` on the state; and `bind : H a p q → (x:a → H b (q x) r) → H b p r`, a combinator whose indexing structure represents the rule for sequential composition in Hoare logics. Encoding a Hoare logic in the indices is a powerful concept, and Hoare monads have been used to prove the correctness of many programs in a variety of program logics. But, even among Hoare monads, several variants exist. For example, in some versions, the postcondition `q` is a relation on a pair of memory states, i.e., `q: mem → a → mem → prop`.

*Dijkstra monads.* Seeking to compute verification conditions for programs with effects beyond just state, Swamy et al. [2013] proposed Dijkstra monads. Refined further by Swamy et al. [2016], Ahman et al. [2017] and Maillard et al. [2019], a Dijkstra monad `D` is a monad-like typeclass where the indexing structure is itself a monad {`M`, `return`, `bind`}. That is, `D` has the following combinators: `return: x:a → D a (M.return x)`, and `bind: D a m → (x:a → D b (n x)) → D b (M.bind m n)`. Intuitively, the computational monad `D` is abstracted by the specification monad `M`, with a morphism between the two encoded in the indexing structure. Dijkstra monads are at the core of the F$^\star$ programming language [Swamy et al. 2016] and have been used in the verification of several large developments [Bhargavan et al. 2017].

### 1.1 New hybrid constructions

A central observation of this paper is that while each of these indexed monad structures offer reasoning principles for effectful programs on their own, using them in combination yields a multitude of other structures that can help in producing simpler, more structured proofs of effectful programs. We present new hybrid constructions involving graded Hoare monads, parameterized Dijkstra monads, graded Dijkstra monads, graded doubly-Hoare monads, and other such hybrid structures, exploiting them to simplify the proofs of programs ranging from the correctness of binary format serializers to information flow control.

A starting point to effectively exploit these exotic indexed structures for programming and proving is a unified, programmable framework supporting them all. The syntactic overhead of programming directly with monadic structures is prohibitive–consider that programming with even regular monads is tedious absent Wadler's classic `do`-notation. However, whereas all monadic programs benefit from the `do`-notation, other indexed monads, not being instances of the monad typeclass, do not enjoy such benefits. E.g., in Haskell, parameterized monads are captured by the `Monadish` typeclass,[2] for which no special syntax is available. Given the diversity of indexing structures we wish to use, a single typeclass to cover them all is infeasible.

### 1.2 Type-and-effect directed elaboration

Lacking a typeclass for our structures, we develop a new language feature to support a type-and-effect directed elaboration of source programs written in a direct, applicative syntax into any indexed monad structure. Our feature, *indexed effects*, is usable with any monad-like type constructor `L`, with an arbitrary number of indices, and combinators `return : x:a → L a` $\vec{i}$ and `bind : L a` $\vec{i}$ `→ (x:a → L b` $\vec{j}$`) → L b` $\vec{k}$—note the indices may vary arbitrarily in `return` and `bind`. Given such a signature, our algorithm allows programs to be developed in an ML-like applicative syntax, while elaborating them automatically into the underlying monad-like combinators on `L`. We have implemented indexed effects in F★, enhancing its user-defined effect system, previously limited to Dijkstra monads only, to cover all forms of indexed monads.

For a first example, consider the graded monad `gst (a:Type) (t:tag)`, with `tag = R | RW` shown below, with a refinement type to state that read-only computations do not modify the state, and with actions to `read` and `write` the state.

```
let state = int      type tag = R | RW      let (⊕) t0 t1 = match (t0, t1) with | (R, R) → R | _ → RW
let gst (a:Type) (t:tag) = s0:state → r:(a & state) { t=R ⟹ s0 == snd r }
let return (x:a) : gst a R = λs → x,s
let bind (f:gst a t0) (g: a → gst b t1) : gst b (t0 ⊕ t1) = λs0 → let x, s1 = f s0 in g x s1
let read () : gst state R = λs → s,s      let write (s:state) : gst unit RW = λ_ → (), s
```

To increment the state, one would write `bind (read()) (λx → write (x + 1))` and many dependent type systems could infer the type `gst unit RW`. For such a simple program, this may seem adequate. However, as the indices become richer, explicitly monadic programming can be an obstacle.

With our new support for user-defined indexed effects in F★, we can turn the `gst` monad into a new indexed *computation type* `GST`, while also indicating to the system to implicitly re-index types when needed, e.g., in the branches of conditional computations.

```
let subcomp (f:gst a t0 { ∃t. t1 == t0 ⊕ t }) : gst a t1 = f
let if_then_else (f:gst a t0) (g:gst a t1) (_:bool) = gst a (t0 ⊕ t1)
effect { GST (a:Type) (t:tag) with { repr = gst; return; bind; read; write; subcomp; if_then_else } }
```

With these definitions in place, one can write `if b then (write (read () + 1); 0) else read()`, while the framework infers the computation type `GST int RW` and internally elaborates the program into the following explicitly monadic form:

```
if b then subcomp (bind (read()) (λ x → bind (write (x + 1)) (λ _ → return 0))) else subcomp (read())
```

Effect definitions can also be layered, e.g., we could add a layer to represent exceptions on top of the `GST` effect, with implicit coercions to move between them.

### 1.3 Formalization of indexed effects and simplification to the theory of F★

To formalize our system, we design Indexed Monadic F★ (IMF★), a surface language with user-defined indexed effects, and a simple type-and-effect directed elaboration of IMF★ programs into TotalF★, a core lambda calculus with dependent and refinement types. Our main theorem proves that the translation from IMF★ to TotalF★ is well-typed (§3).

Prior to our work, the core calculus of F★ included a primitive notion of Dijkstra monads [Ahman et al. 2017; Swamy et al. 2016]. Indeed, all other Dijkstra monads in F★ built upon this primitive notion. With IMF★, Dijkstra monads can be defined as just another indexed effect and need no

[2] https://hackage.haskell.org/package/twilight-stm-1.2/docs/Control-Concurrent-STM-Monadish.html

longer be primitive. As result, not only does our work add support for programming with rich indexing structures in F$^\star$, but it also simplifies the core logical underpinnings of F$^\star$ to just TotalF$^\star$. Simplifying the core is a significant advancement for a proof assistant.

### 1.4 Applications of the new hybrid constructions

We present three case studies of indexed effects at work. The first is a new graded, Hoare monad for information flow control and functional correctness of stateful programs (§2). Next, we show a library of generic algebraic effects and handlers, using a graded Dijkstra monad to verify heap-manipulating programs in this framework (§4). Finally, we improve support for low-level message formatting in EverParse [Ramananandro et al. 2019], a library of monadic parser and formatter combinators. By layering two parameterized monad-indexed monads on top of EverParse's existing use of a Hoare monad for C programs, we obtain more concise programs and better proof automation, while yielding verified C code devoid of performance overhead (e.g., due to intermediate allocations and copies) inherent in the purely functional formatters developed previously (§5)[3].

Separately, providing evidence of the usefulness of our work at scale, indexed effects in F$^\star$ have already been used extensively in Steel [Fromherz et al. 2021; Swamy et al. 2020], a dependently typed, concurrent separation logic shallowly embedded in F$^\star$, using an indexed monad with six indices to capture various components of Steel's logic. Additionally, Bhargavan et al. [2021] use indexed effects in F$^\star$ to reason about properties of a global, interleaved execution trace of cryptographic protocol sessions, using it at the core of a system that partially automates symbolic proofs of cryptographic security (§6).

In all these cases, effect indices provide an abstraction to reason about effectful programs. When these indices come from familiar algebraic structures like monoids and monads, proofs of effectful programs can be reduced to purely functional programming, following Xavier Leroy's guidance.

The diversity of our experience encourages us to conclude that richly indexed effects, coupled with simple language support for elaboration, allows program proof developers to craft new abstractions and benefit from simpler proofs, while also enjoying a direct programming style with automatic inference and elaboration into an small, core calculus of pure computations. We hope that a unifying framework like ours will make it easier for the programmers to adopt and benefit from the great many indexing structures from the literature.

## 2 INDEXED EFFECTS IN F$^\star$, BY EXAMPLE

This section introduces indexed effects in F$^\star$ progressively, starting with a simple, non-indexed state monad and working our way eventually to a graded, 2-state Hoare monad for functional correctness proofs for stateful programs. We emphasize two points:

- By carefully designing the indexing structure on a monadic effect, it is possible to reason about programs in an abstraction well-suited to the reasoning task at hand.
- Regardless of the indexing structure, e.g., whether no indices are used at all, or if the indices are drawn from some rich logic, our type-and-effect directed elaboration helps in hiding the complexity of the underlying semantic models of an effect from a programmer, providing, in addition to syntactic elaboration, features such as automated subsumption and coercion between effects.

### 2.1 Background on F$^\star$ and a non-indexed effect for state

We start with a review of F$^\star$ and show how to define a simple effect based on an ordinary state monad.

[3] We submit all the examples presented in the paper as anonymous supplementary material.

F* is a program verifier and a proof assistant based on a dependent type theory with a countable hierarchy of predicative universes (like Coq or Agda). Proofs in F* are partially automated using the Z3 SMT solver [de Moura and Bjørner 2008], although it also has a metaprogramming system inspired by Lean and Idris (called Meta-F* [Martínez et al. 2019]) that allows using F* itself to build and run tactics for constructing programs or proofs. Rather than focusing on purely functional programming, F* has been used extensively to build security-critical, high-performance, low-level software in several embedded DSLs. The resulting code has been deployed in a variety of settings, including the Windows kernel, the Linux kernel, the Microsoft Azure cloud, the Firefox web browser, and several other applications where high-assurance effectful programs are necessary. In service of these scenarios, an integral part of F* is its ability to be extended with user-defined effects. To date, effects in F* have been tied to Dijkstra monads [Ahman et al. 2017; Maillard et al. 2019; Swamy et al. 2013]—no longer, as we will soon see.

*Basic syntax.* F* syntax is roughly modeled on OCaml (`val`, `let`, `match`, etc.). Binding occurrences `b` take the form `x:t`, declaring a variable `x` at type `t`; or `#x:t` indicating that the binding is for an implicit argument. The syntax $\lambda b_1 \ldots b_n \to t$ introduces a lambda abstraction (metavariable `t` ranges over both types and terms), whereas $b_1 \to \ldots \to b_n \to C$ is the shape of a curried function type with *computation type* `C` (more about them shortly). Refinement types are written `b{t}`, e.g., the type `x:int{x≥ 0}` represents natural numbers. We define `squash t` as the unit refinement `_:unit{t}`, which can be seen as the type of (computationally irrelevant) proofs of `t`. As usual, we omit the type in a binding when it can be inferred; and for non-dependent function types, we omit the variable name. E.g., the type `#a:Type → #m:nat → #n:nat → vec a m → vec a n → vec a (m + n)` represents the append function on vectors, where the two explicit arguments and the return type depend on the three implicit arguments marked with '`#`'. We mostly omit implicit binders, except when needed for clarity, treating all unbound variables in types as prenex quantified, writing the type of append as just `vec a m → vec a n → vec a (m + n)`. We also omit universe annotations.

Returning to the computation types `C`, F* distinguishes computations from values in a manner similar, though not identical, to Levy's (2004) Call-By-Push-Value calculus. Computation types include `Tot t` ($x{:}t_1 \to t_2$ is a shorthand for $x{:}t_1 \to$ `Tot` $t_2$) for pure, total computations. Another built-in computation type is `Lemma (requires p) (ensures q)`, which is the type of a computation which when executed in a context validating `p` terminates in a context validating `q`, i.e., it can be seen as sugar for `squash p → squash q`. When `p` is trivial, we simply write `Lemma (ensures q)` or `Lemma q`.

F* also allows users to define new computation types, however, to date, every user-defined computation type was required to be a Dijkstra monad of predicate transformers, either axiomatized [Swamy et al. 2016] or derived using a CPS transformation of programs in a sub-language of effect definitions [Ahman et al. 2017]. Defining even a simple non-indexed state monad as computation type was not possible, until now.

*A simple state monad and its corresponding effect.* Defining a state monad in F* is easy, just as in many functional languages.

```
let st (a:Type) (s:Type) = s → a & s (* & is the tuple type constructor *)
let return (x:a) s : st a s = λs → x, s
let bind (f:st a s) (g:a → st b s) : st b s = λs → let x, s' = f s in g x s'
let get () : st s s = λs → s, s
let put (x:s) : st unit s = λ_ → (), x
```

One can, of course, write programs like this `bind (get()) (λs → put (s + 1))` to increment the state, and F* will infer the type `st unit int` for it. However, this style quickly becomes cumbersome. While many languages offer a do-notation for monads (F* does too) and in the case of ordinary monads such as this one, the do-notation is adequate, indexed effects provide an alternative which scales also to indexed monads. With indexed effects, F⋆ allows users to define a new effect, using the notation below:

```
effect { ST (a:Type) (s:Type) with { repr = st; return; bind; get; put } }
```

We use the term "effect" to refer to a computation type constructor. Here, `ST` is an effect and the definition above introduces a new user-defined computation type, `ST a s`, whose underlying representation is `st a s`, supporting the `return` and `bind` combinators, and two actions `ST?.get : unit → ST s s` and `ST?.put : s → ST unit s`—the notation `ST?.op` names the operation `op` in the `ST` effect declaration.[4]

With this in place, one can write `ST?.put (ST?.get() + 1)` and have F⋆ infer the type `ST unit int`, while elaborating the program internally to the explicitly monadic notation shown earlier. Since computation types can appear only to the right of an arrow, corresponding to a call-by-value evaluation strategy, and by enforcing left-to-right evaluation order, the elaboration into the explicitly monadic notation becomes algorithmic.

For ordinary monads, this may not seem like much. Indeed, what we have here corresponds closely to a type-and-effect elaboration into explicitly monadic notation developed previously for ML-like programs and ordinary monads by Swamy et al. [2011]. A main contribution of this paper is to show how this basic idea can be generalized to work in a dependently typed setting with indexed monads of all flavors.

## 2.2 Indexed effects, in a nutshell

An effect declaration in F⋆ allows promoting any indexed monad `m` into an effect `M`. Doing so requires:

1. A representation type, `m t` $\bar{i}$, for an arbitrary arity $|\bar{i}|$.
2. A `return`, whose type is of the form `x:a → m a` $\bar{p}$, for some $\bar{p}$.
3. A `bind`, whose type is of the form `m a` $\bar{p}$ → `(x:a → m b` $\bar{q}$`) → m b` $\bar{r}$, for some $\bar{p}$, $\bar{q}$, $\bar{r}$
4. Zero or more actions, $\bar{a}$ where each $a_i$ has a type of the form $\overline{x_i : t_i} \to m\ s_i\ \bar{p}_i$.
5. An optional subsumption combinator, `subcomp, _:m a` $\bar{p}$ `{ pre } → m a` $\bar{q}$, which allows re-indexing an `m a` $\bar{p}$ to an `m a` $\bar{q}$, when `pre : prop` is valid.
6. An optional branching combinator, `if_then_else` whose type has the form `m a` $\bar{p}$ `→ m a` $\bar{q}$ `→ bool → Type`, such that for all `x:m a` $\bar{p}$, `y:m a` $\bar{q}$, and `b:bool`, `if b then subcomp x else subcomp y` has type `if_then_else x y b`.

Having introduced an effect `M` based on `m`, our type-and-effect system infers computation types `M a` $\bar{i}$ for programs using the effectful actions `M?.`$a_i$, and automatically elaborates them into the underlying monadic operations on `m`, while generating verification conditions to show that the inferred type of a program is compatible with a user-provided annotation, implicitly re-indexing terms as needed using `subcomp` and `if_then_else`. The resulting verification conditions can be dispatched in F⋆ using a variety of techniques, ranging from SMT solving to interactive proofs with tactics.

*Composing multiple effects.* Further, given two effect declarations `M` and `N` with representation types `m` and `n`, F⋆'s effect system supports implicitly lifting `M`-computations to `N`-computations if the programmer supplies a combinator `lift: _:m a` $\bar{i}$`{ pre } → n a` $\bar{j}$.

As we will soon see, indexed effects allow one to design multiple, effect-based domain-specific languages in F⋆, and for those languages to be composable, while enjoying the full native syntax of F⋆ (with let bindings, pattern matching, recursion, etc.), verification condition generation, and proof

[4] Strictly speaking, to reference the actions in the GST effect of §1.2, we should have written `GST?.read` and `GST?.write`, though we omitted the "GST?." prefix for simplicity.

automation in F*, with a foundation of trust built within F* upon a model of effectful computations as indexed monads.

### 2.3 A Hoare monad for functional correctness proofs of stateful programs

To prove the correctness of stateful programs, Nanevski et al. [2008] proposed to refine the state monad `st a s = s → a & s` with predicates drawn from a Hoare logic. In our setting, this can be encoded like so:

```
let hst a s (p:s → prop) (q:s → a → s → prop) = s0:s{p s0} → x:(a & s) { q s0 (fst x) (snd x) }
```

The type `hst a s p q` represents a state-passing computation which, when run in an initial state `s0:s` validating the precondition `p s0`, returns a `result:a` and a final state `s1:s` that validates the postcondition `q s0 result s1`. One can define the following combinators.

```
let return (x:a) : hst a s (λ _ → ⊤) (λ s0 r s1 → s0==s1 ∧ r==x) = λs → x, s
let bind (f:hst a s p_f q_f) (g: (x:a → hst b s (p_g x) (q_g x)))
  : hst b s (λ s0 → p_f s0 ∧ (∀ x s1. q_f s0 x s1 ⟹ p_g x s1)) (λ s0 r s2 → (∃ x s1. q_f s0 x s1 ∧ q_g x s1 r s2))
  = λs0 → let x, s1 = f s0 in g x s1
```

This is different from a classic Hoare monad, for a few reasons. First, the postcondition we use here is a predicate covering over both the initial and final state—a so-called, 2-state postcondition. Further, a standard Hoare monad as sketched in Nanevski et al. [2008] has a `bind` with a signature resembling Atkey's parameterized monad in that the postcondition of the `f` matches the precondition of `g`. However, our `bind` for `hst` is designed so that it is parametric in the pre- and postconditions of both `f` and `g`, making type inference and verification condition generation for `hst` easier. To enable this, we must also strengthen the precondition of the resulting computation with a requirement that $q_f$ is stronger than $p_g$, while, to retain precision, the final postcondition is also strengthened with both $q_f$ and $q_g$. Non-standard or not, `hst` can be easily promoted to an effect in F*, since the effect mechanism places no restrictions on the indexing structure. But, before promoting `hst` to an effect, we'll define some actions, a subsumption rule, and a type for composing branching computations.

*Actions for* `hst`. The `get` and `put` actions are computationally equivalent to their unrefined counterparts in `st`. In `hst`, we give them precise logical specifications.

```
let get () : hst s s (λ _ → ⊤) (λ s0 x s1 → s0 == s1 ∧ x == s1) = λs → s, s
let put (x:s) : hst unit s (λ _ → ⊤) (λ _ _ s1 → x == s1) = λ_ → (), x
```

*Subsumption, or the Hoare rule of consequence.* Hoare logics typically include a rule of consequence, enabling preconditions to be strengthened and postconditions to be weakened. Our Hoare logic encoded in `hst` also admits such a rule, which we can encode as a subsumption combinator for re-indexing `hst`, shown below.

```
let subcomp (x:hst a p q {relate_pre_post p p' q q'}) : hst a p' q' = x
where relate_pre_post p p' q q' = (∀ s. p' s ⟹ p s) ∧ (∀ s0 x s1. p' s0 ∧ q s0 x s1 ⟹ q' s0 x s1)
```

*Branching.* Whereas `bind` is required to specify how to sequentially compose computations, within our framework, it is also possible to specify how to type and compose computations under branches. In this case, to typecheck `if b then f else g` it suffices to take the join of their types by simply lifting the conditional to the level of the indices.

```
let ite (f:hst a s p_f q_f) (g:hst a s p_g q_g) b = hst a s (if b then p_f else p_g) (if b then q_f else q_g)
```

Finally, an effect definition promotes `hst` to `HST`, as shown below.

```
effect { HST (a:Type) (s:Type) (p:s → prop) (q:s → a → s → prop) with
        { repr = hst; return; bind; subcomp; if_then_else = ite; get; put }}
```

To write and prove stateful programs in HST, one starts by picking a model for mutable memory. To illustrate, we choose just a store that maps natural number memory locations (loc) to integers, where Map.t is a total map from F*'s standard library, supporting operations to select (sel) and update (upd) keys in the map. We build derived actions to read and write locations in memory, giving them precise specification in HST. We do not model dynamic allocation nor typed references, as they are orthogonal—several other memory models in F* support such features [Ahman et al. 2018; Protzenko et al. 2017; Swamy et al. 2020] and are usable with indexed effects.

```
let loc = nat        let store = Map.t loc int
let read (x:loc) : HST int store (λ _ → ⊤) (λ s0 v s1 → s0 == s1 ∧ v = sel s1 x) = sel (HST?.get ()) x
let write (x:loc) (v:int) : HST unit store (λ _ → ⊤) (λ s0 _ s1 → s1 == upd s0 x v)
  = HST?.put (upd (HST?.get()) x v)
```

Putting several features together, we implement and prove programs like so, where we tag the pre- and postcondition with the (semantically irrelevant) keywords requires and ensures just to improve readability:

```
let mod_or_sqr (b:bool) (x y:loc) : HST unit store
  (requires λs → sel s x > 0)
  (ensures λs0 _ s1 → ∃v. (if b then v ≤ sel s0 x else v ≥ sel s0 x) ∧ s1 == upd s0 y v)
  = if b then write y (read y % read x) else write y (read x * read x)
```

For even this simple program, the type-and-effect based elaboration is a significant benefit. First, in each branch of the condition, we have imperative code developed directly in an applicative notation, rather than requiring it to be explicitly monadic. Next, although each branch has a different type, the if_then_else combinator provides a form of dependent pattern matching, automatically giving the conditional a type that depends on the branch condition b. Finally, when the user annotates a specification for a function, the system automatically applies the rule of consequence, building a verification condition, which, in this case, is automatically discharged by SMT.

Without the effect-based elaboration, one could try to write a program directly in the hst monad. It would look something like this (where read_hst and write_hst are the hst analogs of read and write), where one essentially has to build a Hoare-style derivation by hand. Even if the system is able to infer all the missing implicit arguments (shown as underscores), which it cannot, in this case, this style is verbose and obscures the program beyond recognition.

```
let mod_or_sqr (b:bool) (x:loc) (y:loc)
  : hst unit store (λ s → sel s x > 0)
                   (λ s0 _ s1 → ∃v. (if b then v ≤ sel s0 x else v ≥ sel s0 x) ∧ s1 == upd s0 y v)
  = subcomp _ _ _ _ _ _
     (if b then subcomp _ _ _ _ _ _
                   (bind _ _ _ _ _ _ _ (read_hst y) (λ v0 →
                    bind _ _ _ _ _ _ _ (read_hst x) (λ v1 →
                    bind _ _ _ _ _ _ _ (lift_pure_hst _ _ (v0 % v1)) (λ i →
                    write_hst y i))))
      else subcomp _ _ _ _ _ _
                   (bind _ _ _ _ _ _ _ (read_hst x) (λ v0 →
                    bind _ _ _ _ _ _ _ (read_hst x) (λ v1 →
                    write_hst y (v0 * v1)))))
```

*Sub-effects.* If you look closely at the elaboration above, you may wonder about `lift_pure_hst` in the `then`-branch. What's happened here is that our system has automatically lifted a pure computation with a non-trivial precondition into the `hst` effect. We explain this in greater detail in §3.4, summarizing the main idea here. At the core of F*, pure computations with non-trivial preconditions are typed in a Dijkstra monad `PURE a wp`, a type for *conditionally pure* computations which when evaluated in a context validating `wp p : prop`, for any `p : a → prop`, return a `v : a` validating `p v`. In this case, we have a term `v0%v1` which, due to a possible division by zero, has type `PURE int (λp → v1 ≠0 ∧ (v1≠0 ⟹ p (v0 % v1)))`, indicating that it is only safe run the term in context where `v1 ≠0` is valid. To incorporate such conditionally pure computations within `HST`, one can specify that `PURE` is a *sub-effect* of `HST`, as shown below.

```
let pure a (wp : (a → prop) → prop) = unit → PURE a wp
let lift_pure_hst (f : pure a wp) : hst a
  (requires λ_ → wp (λ _ → ⊤))
  (ensures λs0 v s1 → s0 == s1 ∧ ¬(wp (λ y → ¬(y == v))))
  = λs → f(), s
sub_effect PURE ⇝HST = lift_pure_hst
```

Our system maintains a directed acyclic graph of sub-effects, ensuring that two effects are related by sub-effecting in at most one way. During elaboration, this allows us to unambiguously lift one effect to another, while relating the indexing abstractions appropriately. In this case, `lift_pure_hst` interprets the predicate transformer index `wp` as a Hoare-style precondition (applying it to a trivial postcondition), and as a postcondition, relying on a double-negation transformation of the `wp`.

From even this simple example, we hope to illustrate that due to the prohibitive syntactic overhead, novel indexed monad constructions, despite providing very useful abstractions for program reasoning, are difficult to adopt in practice. Instead, with our effect elaboration system, one can freely explore the design space of indexed monads to build suitable abstractions applicable to programs that humans can write, understand, and prove correct, with good automation. As an instance of such an exploration, we present next a novel refinement of `HST`, indexing it additionally with a non-trivial monoid to track read and write effects and information flows, while still benefiting from automated elaboration.

### 2.4 Refining `HST` with information flow control

While the Hoare logic encoded in the `HST` effect is expressive enough for functional correctness proofs, the specifications it permits are relatively unstructured—pre- and postconditions are just predicates on the entire store—and are limited to properties of a single execution of a program. In this section, we present `HIFC`, a refinement of `HST`, based on a graded Hoare monad for state, where computation types of the form `HIFC a reads writes flows pre post` constrain the set of memory locations read and written, and the dependences among them, in addition to the Hoare-style pre- and postconditions. We summarize our construction here, with the full details in the appendix.

To define the effect `HIFC`, we'll start with a indexed monad representation, `hifc`, refining `hst`.

```
let label = set loc        let flow = label & label        let flows = list flow
let hifc a (r w:label) (fs:flows) p q = f:hst a store p q { writes f w ∧ reads f r ∧ respects f fs }
```

Interpreting the write index, `writes f w`, states that all locations not in `w` are unchanged when running `f` in any state. The `reads` predicate involves a relational interpretation, similar to Benton et al. [2006], stating that runs of `f` on stores that differ only on unread locations are equivalent. The `respects` relation is the main statement of noninterference, also stated relationally—information flows from `l` to `m` only if there exists some `(src, dest) ∈ flows` such that `l ∈ src` and `m ∈ dest`.

Next, we define `return`, to lift pure terms into the `hifc` abstraction; and `bind` to show that the abstraction of `hifc` is stable under sequential composition. The proof of the correctness of `bind` is non-trivial and requires about a few hundred lines of auxiliary lemmas in F★, but this proof is done once and for all. We just show the signatures.

```
val return (x:a) : hifc a {} {} [] (λ _ → ⊤) (λ s0 r s1 → s0 == s1 ∧ r == x)

val bind (f:hifc a r0 w0 fs0 p q) (g: (x:a → hifc b r1 w1 fs1 (r x) (s x)))
  : hifc b (r0 ∪ r1) (w0 ∪ w1) (fs0 @ add_source r0 (({}, w1)::fs1))
           (λ s0 → p s0 ∧ (∀ x s1. q s0 x s1 ∧ modifies w0 s0 s1 ⟹ r x s1))
           (λ s0 r s2 → (∃ x s1. (q s0 x s1 ∧ modifies w0 s0 s1) ∧ (s x s1 r s2 ∧ modifies w1 s1 s2)))
```
where `add_source r fs = List.map (λ (src, sink) → (src ∪ r, sink)) fs` and `@` is list concatenation

The type of `bind f g` has several interesting elements. `bind f g` reads (and writes) the union of the read (and write) sets of `f` and `g`. More subtly, the flows of `bind f g` are the flows of `f` (`fs0`), together with the flows of `g` (`({}, w1)::fs1`) augmented with flows from the locations read by `f` (`r0`), since `g`'s argument is tainted by `f`'s reads. This way of computing flows is more precise than in prior monadic IFC systems, which usually consider that all locations read by `f` can flow to all locations written by `g`. Finally, showing the use of the write index for framing, both the pre- and postcondition exploit the invariant that `f` does not modify locations outside `w0` and `g` outside `w1` (where the predicate `modifies w s0 s1` states that `s0` and `s1` agree on all locations outside `w`.). As such, the `write` index encodes a form of dynamic framing [Kassios 2006].

We can show that the triple of additional indices of `HIFC`, (`reads`, `writes`, `flows`) form a monoid (under a suitable equivalence relation) whose unit is {}, {}, [] and whose elements can be composed with ⊕ (shown below), which is the indexing pattern of a graded monad, used on `return` and `bind`.

$$(r_0, w_0, f_0) \oplus (r_1, w_1, f_1) = r_0 \cup r_1, w_0 \cup w_1, f_0 \mathbin{@} \mathtt{add\_source}\ r_0\ ((\{\}, w_1)::f_1)$$

Packaging `hifc` as an effect `HIFC`, together with a subsumption relation that allows widening the reads, writes and flows sets together with the Hoare rule of consequence; a branching combinator; and actions to read and write individual locations, allows us to write and prove effectful programs for both correctness and security, by reasoning only about their indices. For example, $\mathtt{write}\ l_1 (\mathtt{read}\ l_0)$ is inferred to have type $\mathtt{HIFC\ unit}\ \{l_0\}\ \{l_1\}\ [(\{l_0\}, \{l_1\})]\ (\lambda\_ \to \top)\ (\lambda s_0\ \_\ s_1 \to \mathtt{sel}\ s_1 l_1 == \mathtt{sel}\ s_0 l_0)$.

*Refining flows with Hoare reasoning.* Monadic label-based information flow is inherently imprecise, since it conflates data and control dependence. To illustrate, consider `read h; write l (read l + 1)` which has the type $\mathtt{HIFC\ unit}\ \{h, l\}\ \{l\}\ [(\{h\}, \{l\})]\ (\lambda\_ \to \top)\ (\lambda s_0\ \_\ s_1 \to \mathtt{sel}\ s_1 l = \mathtt{sel}\ s_0 l + 1)$, suggesting that it leaks information from `h` to `l`, when in reality no such flow exists since the `read h` is redundant. However, `HIFC`'s pre- and postconditions can be used to recover precision. The re-indexing coercion, `refine`, allows removing a flow `f` from `HIFC a r w (f::fs) p q` when the Hoare specifications `p` and `q` allow proving that the `f`-flow is spurious.

```
val refine (_: (unit → HIFC a r w (f::fs) p q) {(∀ from to v. from ∈ fst f ∧ to ∈ snd f ∧ from ≠ to ⟹
  (∀ s0 x x' s1 s1'. (p s0 ∧ p (upd s0 from v) ∧ q s0 x s1 ∧ q (upd s0 from v) x' s1' ∧
              modifies w s0 s1 ∧ modifies w (upd s0 from v) s1') ⟹
              sel s1 to == sel s1' to))}) : unit → HIFC a r w fs p q
```

Using `refine` (which the programmer must explicitly apply), we can revise the type of our example term to `HIFC unit {h, l} {l} [] ...`, removing the spurious flow.

We have seen how in the type of `bind` the Hoare specifications are improved using the `write` index by internalizing framing. Conversely, with `refine`, Hoare specifications also improve the precision of the information flow labels, illustrating the useful interplay between the two indexing

| | | | | | | |
|---|---|---|---|---|---|---|
| constant | $T$ | ::= | unit \| Type$_u$ \| sum \| inl \| inr | computation type | $C$ | ::= Tot $t$ |
| term | $e, t$ | ::= | $x \mid T \mid x{:}t_1\{t_2\} \mid x{:}t \to C \mid \lambda x{:}t.\ e \mid e_1\ e_2 \mid \mathsf{case}_C(e \text{ as } y)\ x.e_1\ x.e_2$ | | | |

Fig. 1. Syntax of TotalF$^\star$

structures. Our point is that by designing hybrid indexed monads, one can obtain new reasoning principles whose combination is more powerful than their components.

## 3 FORMALIZATION AND IMPLEMENTATION OF INDEXED EFFECTS

In this section we provide a semantics for indexed effects via a type-and-effect directed, type-preserving translation to a core lambda calculus with dependent and refinement types. The essence of the translation is that it unfolds a computation type into its underlying indexed monadic representation. As a result, while the programmers may use indexed effects to build abstractions for the ease of programming and proving, the structure of types still provides guidance as to how the programs execute.

This translation semantics of indexed effects also allows us to minimize the formal core of F$^\star$. Specifically, we can remove all the effectful features from it, including currently primitive Dijkstra monads, as they can simply be encoded. This in itself is a significant advancement, since any further metatheory of F$^\star$'s core no longer has to consider Dijkstra monads. The translation semantics may also provide a way for other proof assistants, many of which have core calculi resembling TotalF$^\star$, to implement indexed effects, though some of the details, like refinement types, may differ.

We begin by describing the target language of the translation, and the new core calculus of F$^\star$, that we call TotalF$^\star$. We then present Indexed Monadic F$^\star$ (IMF$^\star$), a surface language with user-defined indexed effects, and present a syntax-directed, type-and-effect driven translation from IMF$^\star$ to TotalF$^\star$; we prove that the translation is type-preserving implying that the typing derivations in IMF$^\star$ can be soundly interpreted in TotalF$^\star$. We also discuss several implementation aspects of indexed effects in the F$^\star$ typechecker.

### 3.1 TotalF$^\star$: The new core calculus for F$^\star$

Figure 1 shows the syntax of TotalF$^\star$. It includes proof-irrelevant refinement types, dependent functions, dependent pattern matching with case (with explicitly annotated return computation type $C$), and a non-cumulative predicative universe hierarchy (Type$_u$). We elide universe annotations in the rest of the section. TotalF$^\star$ has one computation type Tot $t$ for total computations. The calculus is essentially an effectless fragment of EMF$^\star$ [Ahman et al. 2017], the previous F$^\star$ core. Though terms and types are in the same syntactic class in TotalF$^\star$, we use $e$ and $t$ to distinguish them, when it promotes clarity. Similarly, we use $x$ as term variables, and $a$ and $b$ as type variables.

The most distinctive feature of TotalF$^\star$ is its use of a subtyping relation in type conversion. Definitionally equal types are convertible. Subtyping also includes types related by refinement, where $\Gamma \vDash t$ is a proof-irrelevant logical entailment, implemented in F$^\star$ using an SMT solver, or handled by user-provided tactics.

$$\frac{t_0 \longrightarrow^* t \qquad t_1 \longrightarrow^* t}{\Gamma \vdash t_0 <: t_1} \qquad \frac{\Gamma \vdash t_0 <: t_0' \qquad \Gamma, x{:}t_0\{t_1\} \vDash t_1'}{\Gamma \vdash x{:}t_0\{t_1\} <: x{:}t_0'\{t_1'\}}$$

TotalF$^\star$ is still simpler than the logical core of the full F$^\star$ system, lacking inductive type definitions and general pattern matching, recursive functions, and universe polymorphism. Importantly, TotalF$^\star$ lacks F$^\star$'s equality reflection, which is crucial to F$^\star$'s extensional type theory. We plan to

$$
\begin{array}{llcl}
\text{constant} & T & ::= & \mathsf{unit} \mid \mathsf{Type}_u \mid \mathsf{sum} \mid \mathsf{inl} \mid \mathsf{inr} \\
\text{term} & e, t & ::= & x \mid T \mid x{:}t_1\{t_2\} \mid x{:}t \to C \mid \lambda x{:}t.\ e \mid e_1\ e_2 \mid \mathsf{let}\ x\ =\ e_1\ \mathsf{in}\ e_2 \\
 & & \mid & \mathsf{case}_C(e\ \mathsf{as}\ y)\ x.e_1\ x.e_2 \mid \mathsf{reify}\ e \mid F.\mathsf{reflect}\ e \\
\text{computation type} & C & ::= & \mathsf{Tot}\ t \mid F\ t\ \overline{e} \qquad \text{effect label} \quad M\ ::= \mathsf{Tot} \mid F \\
\text{effect definition} & D & ::= & F\ (a : \mathsf{Type})\ \overline{(x : t)}\ \{e_{repr}; e_{return}; e_{bind}; e_{subcomp}\} \\
\text{lift definition} & L & ::= & \mathsf{lift}_{M_1}^{M_2} = e \qquad \text{signature} \quad S\ ::= \cdot \mid D, S \mid L, S \\
\text{type environment} & \Gamma & ::= & \cdot | x : t, \Gamma \qquad \text{typing context} \quad \Delta\ ::= S; \Gamma
\end{array}
$$

Fig. 2. Syntax of IMF★

study these enhancements in the future, grateful to no longer have to consider their interaction with Dijkstra monads.

### 3.2 IMF*: A surface language with user-defined indexed effects

IMF★ models a surface language with support for user-defined indexed effects. Figure 2 shows the syntax. IMF★ adds effectful constructs to TotalF★. These include computation types $F\ t\ \overline{e}$ (where $F$ is the effect, $t$ is the return type, and $\overline{e}$ are the effect indices) in $C$, let bindings, and reify and reflect coercions between the computation types and their underlying representations.

An effect definition $D$ defines an indexed effect $F$ with indices types $\overline{x : t}$ and combinators $e_{repr}$, $e_{return}$, $e_{bind}$, and $e_{subcomp}$, while $\mathsf{lift}_{M_1}^{M_2}$ defines a combinator to lift $M_1$ computations to $M_2$ (effect $M$ ranges over Tot and $F$). Our implementation allows for specifying an optional custom effect combinator for combining the branches of case–we discuss it in §3.5.

IMF★ inherits the rest from TotalF★. Thus, the monadic structure of the terms is implicit in the surface syntax, and is elaborated into explicit binds and lifts by the typing judgment.

The main typechecking judgment in IMF★ has the form $\Delta \vdash e : C \rightsquigarrow e'$ stating that under a typing context $\Delta$, expression $e$ has computation type $C$ and elaborates to expression $e'$ in TotalF★. When the elaborated term is not significant, we just write $\Delta \vdash e : C$.

To illustrate the typing rules, we use the graded state monad from Section 1, partly reproduced here, as a running example. One difference, to illustrate the use of `reflect`, is that rather than including the `read` and `write` actions as part of the effect definition, we show a desugared form where we use `GST.reflect` to promote a `gst a t` to a `GST a t` computation type, separately from the effect definition.

```
let gst (a:Type) (t:tag) = s0:state → r:(a & state) { t=R ⟹ s0 == snd r }
let return a (x:a) : gst a R = λs → x,s
let bind a b t0 t1 (f:gst a t0) (g: a → gst b t1) : gst b (t0 ⊕ t1) = λs0 → let x, s1 = f s0 in g x s1
effect { GST (a:Type) (t:tag) with { repr=gst; return; bind } }
let read () : GST state R = GST.reflect (λ s → s,s)
let write s : GST unit RW = GST.reflect (λ _ → (), s)
```

*Typechecking effect definitions and lifts.* While IMF★ does not impose any constraints on the layering or indexing structure of the effects, the types of the combinators in an effect definition $D$ are constrained to have specific shapes, as described in §1.2. Whereas previously we left the additional index arguments in these combinators implicit, here, to be clearer, we make them explicit. We write $F.repr$, $F.bind$ etc. to denote $e_{repr}$, $e_{bind}$ etc. for the effect $F$ in an ambient signature $S$.

An effect definition for $F$ is typechecked as follows. $F.repr$ has a function type with argument types matching the effect signature $F\ (a : \mathsf{Type})\ \overline{x : t}$. $F.return$ and $F.bind$ have monad-like shapes

$$\textsc{T-Var}\ \frac{x : t \in \Gamma}{\Delta \vdash x : t \rightsquigarrow x}
\qquad
\textsc{T-Let}\ \frac{\begin{array}{c}\Delta \vdash e_1 : C_1 \rightsquigarrow e_1' \qquad \Delta \vdash \lambda x.\, e_2 : x{:}C_1^t \to C_2 \rightsquigarrow e_2' \qquad x \notin FV(C_2^t) \\ \Delta, f : \hat{C}_1, g : x{:}C_1^t \to \hat{C}_2 \vdash C.\mathit{bind}\ (\mathsf{lift}_{C_1}^{C}\ f)\ (\lambda x.\ \mathsf{lift}_{C_2}^{C}\ (g\ x)) : \hat{C} \rightsquigarrow e'\end{array}}{\Delta \vdash \mathsf{let}\ x\ =\ e_1\ \mathsf{in}\ e_2 : C \rightsquigarrow e'[e_1'/f][e_2'/g]}$$

$$\textsc{T-Reify}\ \frac{\Delta \vdash e : C \rightsquigarrow e'}{\Delta \vdash \mathsf{reify}\ e : \hat{C} \rightsquigarrow e'}
\qquad
\textsc{T-Reflect}\ \frac{\Delta \vdash e : \hat{C} \rightsquigarrow e'}{\Delta \vdash F.\mathsf{reflect}\ e : C \rightsquigarrow e'}
\qquad
\textsc{T-Sub}\ \frac{\Delta \vdash e : C' \rightsquigarrow e' \qquad \Delta \vdash C' <: C}{\Delta \vdash e : C \rightsquigarrow e'}$$

$$\textsc{T-Case}\ \frac{\begin{array}{c}\Delta \vdash e : \mathsf{sum}\ t_1\ t_2 \rightsquigarrow e' \quad \Delta, y : \mathsf{sum}\ t_1\ t_2 \vdash C : \mathsf{Type} \rightsquigarrow t' \quad i \in 1, 2 \quad T = \mathsf{inl}\ \text{if}\ i = 1, \mathsf{inr}\ \text{o/w} \\ \Delta, x : t_i \vdash e_i : C_i[T\ x/y] \rightsquigarrow e_i' \quad \Delta, x : t_i, f_i : \hat{C}_i[T\ x/y] \vdash \mathsf{lift}_{C_i}^{C}\ f_i : \hat{C}[T\ x/y] \rightsquigarrow e_i''\end{array}}{\Delta \vdash \mathsf{case}_C(e\ \mathsf{as}\ y)\ x.e_1\ x.e_2 : C[e/y] \rightsquigarrow \mathsf{case}_{t'}(e'\ \mathsf{as}\ y)\ x.e_1''[e_1'/f_1]\ x.e_2''[e_2'/f_2]}$$

$$\textsc{C-M}\ \frac{\Delta \vdash \hat{C} : \mathsf{Type} \rightsquigarrow t'}{\Delta \vdash C : \mathsf{Type} \rightsquigarrow \mathsf{Tot}\ t'}
\qquad
\textsc{SC-M}\ \frac{\begin{array}{c}\Delta \vdash C_1 : \mathsf{Type} \rightsquigarrow _ \\ \Delta \vdash \hat{C} <: \hat{C}_1\end{array}}{\Delta \vdash C <: C_1}
\qquad
\textsc{S-Sub}\ \frac{\begin{array}{c}\Delta \vdash t_1 : \mathsf{Type} \rightsquigarrow t_1' \quad \Delta \vdash t_2 : \mathsf{Type} \rightsquigarrow t_2' \\ \Delta \vdash t_1' <: t_2'\end{array}}{\Delta \vdash t_1 <: t_2}$$

Fig. 3. IMF$^\star$ typing judgments (primed symbols are TotalF$^\star$ syntax)

with unconstrained $\overline{x : t}$ binders that may appear in the index terms $\overline{e_f}$, $\overline{e_g}$, and $\overline{e}$. We return to the $F.\mathit{subcomp}$ combinator and the related `if_then_else` combinator later.

$$\begin{array}{lcl}
S; \cdot \vdash F.\mathit{repr} & : & a : \mathsf{Type} \to \overline{x : t} \to \mathsf{Type} \\
S; \cdot \vdash F.\mathit{return} & : & a : \mathsf{Type} \to v : a \to \overline{x : t} \to F.\mathit{repr}\ a\ \overline{e} \\
S; \cdot \vdash F.\mathit{bind} & : & a\ b : \mathsf{Type} \to \overline{x : t} \to F.\mathit{repr}\ a\ \overline{e_f} \to (x{:}a \to F.\mathit{repr}\ b\ \overline{e_g}) \to F.\mathit{repr}\ b\ \overline{e}
\end{array}$$

Turning to our example, we have GST.$\mathit{repr}$ = gst, with one effect index `t:tag`. GST.$\mathit{return}$ = return, and in this case we have no additional index arguments. GST.$\mathit{bind}$ = bind, with the $\mathtt{t}_0$ and $\mathtt{t}_1$ arguments to `bind` being the $\overline{x : t}$ binders. When applying these combinators, our implementation relies on F$^\star$'s existing higher-order unifier to compute instantiations of the $a, b$ type arguments, and the $\overline{x : t}$ arguments.

Finally, an expression $e$ defining a lift from $F$ to $F'$ is typechecked as a coercion from $F.\mathit{repr}$ to $F'.\mathit{repr}$, i.e., $S; \cdot \vdash e : a : \mathsf{Type} \to \overline{x : t} \to F.\mathit{repr}\ a\ \overline{e_f} \to F'.\mathit{repr}\ a\ \overline{e}$. Every user-defined effect in IMF$^\star$ gets an automatic lift from Tot: $\mathsf{lift}_{\mathsf{Tot}}^{F} = F.\mathit{return}$. We discuss checks we impose on the lifts collectively to ensure coherence in §3.5.

### 3.3 Type-and-effect directed elaboration

The typing rules from Figure 3 elaborate the implicitly monadic IMF$^\star$ terms to TotalF$^\star$ by inserting binds and lifts. The placement of these combinators are purely syntax-directed, since computation types can only appear immediately to the right of an arrow, and because we enforce left-to-right evaluation order. However, applying the `bind` and `lift` combinators requires inference of the effect indices and the combinator arguments (e.g., for the $\overline{x : t}$ binders in the combinator types). For this, the system includes a declarative, non-coercive subtyping rule, and implicit arguments in all the rules are chosen nondeterministically. In §3.5, we discuss how this nondeterminism is resolved using F$^\star$'s higher-order unifier and annotation-driven subtyping algorithm.

We adopt some notational conventions. We use $C^t$ to project the return type from a $C$. We hide the context $\Delta$ from the lookup notations when it is clear from the context. $\hat{C}$ is the underlying representation of $C$, defined as $t$ when $C = \mathsf{Tot}\ t$ and $F.repr\ t\ \bar{e}$ when $C = F\ t\ \bar{e}$. We also elide the type $t$ from $\lambda x{:}t.\ e$ when it is clear from the context. $C.bind$ looks up the bind combinator for the effect of $C$ (for $C = \mathsf{Tot}\ \_$, bind desugars to function application). Finally, we write $\mathsf{lift}_{C_1}^{C_2}$ to mean the lift combinator $\mathsf{lift}_{M_1}^{M_2}$ in $S$, where $M_i$ is the effect of $C_i$.

Rule T-Var is the standard variable typing rule, elaborated as is to TotalF⋆. Rule T-Let is the let-binding rule. Whereas EMF* had explicit monadic binds and lifts in the syntax, in IMF⋆, monadic elaboration is type-directed and more accurately describes F⋆'s implementation. The rule first typechecks $e_1{:}C_1$ and $e_2{:}C_2$. Since $C_1$ and $C_2$ could have different effects, the rule lifts them to a common computation type $C$, and binds the resulting $C$ computations. The rule is reminiscent of Swamy et al.'s (2011) and Hicks et al.'s (2014) monadic elaboration rules, though both their calculi are non-dependent. Concretely, the rule introduces two fresh variables $f : \hat{C}_1$ and $g : x{:}C_1^t \rightarrow \hat{C}_2$, applies the lift combinators to $f$ and $g$, and then applies the resulting computations to $C$.bind. The let-binding is assigned the computation type $C$ and the compiled TotalF⋆ term is $e'$ with $e_1'$ and $e_2'$ substituted for $f$ and $g$.

T-Reify and T-Reflect move back-and-forth between a computation type and its representation. Interestingly, the elaboration of reify $e$ (resp. M.reflect $e$) is just the elaboration of $e$; reify and reflect are just identity coercions in IMF⋆ with no counterpart necessary in TotalF⋆. In contrast, Filinski [1999] uses monadic reflection to structure the compilation of monadic computations using state and continuations—we leave exploring this possibility to the future, which may allow for more efficient compilation of user-defined effects.

We now return to the `GST` increment example from Section 1 and show the typing rules at work. To elaborate `let x = read () in write x+1`, the rule T-Let first typechecks `read ():GST state R` and elaborates it to `(λs → s,s)`[5] (the elaboration uses the rules for application and lambda forms which we present in the supplementary material; the rules are straightforward and descend into their subterms as expected). Note that the definition of `read` uses `reflect`, which is an identity in TotalF⋆. Next, the rule typechecks `λx → write x+1:state → GST unit RW` and elaborates it to `(λx _ → (), x+1)`. Since the two effect labels are the same, and `GST.bind` is already a TotalF⋆ term, the final elaborated term is `GST.bind (λs → s,s) (λx _ → (), x+1)`.

Rule T-Case is similar to T-Let. It first typechecks the scrutinee $e$ and the two branches $e_1$ and $e_2$ under appropriate assumptions. The rule then lifts the two branches to $C$ by applying the lift combinators to fresh variables $f_1$ and $f_2$, and constructs the final TotalF⋆ term with appropriate substitutions, as in T-Let.

The rule T-Sub applies subtyping on computations. Rule C-M typechecks a computation type $C$ by typechecking $\hat{C}$. The computation-type subtyping rule SC-M delegates to subtyping on the underlying representations, with any preconditions arising as proof obligations expressed within TotalF⋆'s $\vDash$ entailment relation (dispatched in practice to SMT or to user tactics). Since the rule does not automatically lift $C$, it does not need to be coercive. Similarly, rule S-Sub lifts TotalF⋆'s subtyping rule for use with IMF⋆'s value types.

Our main theorem states that the IMF⋆ translation to TotalF⋆ is type-preserving.

Theorem 3.1. *If* $S; \Gamma \vdash e : C \rightsquigarrow e'$, *then* $S; \Gamma \vdash C : \mathsf{Type} \rightsquigarrow t'$ *and* $[\mid \Gamma \mid]_S \vdash e' : t'$.

Here, $[\mid \Gamma \mid]_S$ is the pointwise translation of the typing environment, and $[\mid \Gamma \mid]_S \vdash e' : t'$ is the typing judgment in TotalF⋆. Using the theorem, a typing derivation in IMF⋆ can be soundly interpreted in TotalF⋆. Ahman et al. [2017] prove EMF* normalizing, type-preserving, and a consistency

[5] The elaborated term is actually `(λ() s → s,s) ()`, we eliminate the application for clarity.

property for its refinement logic—these results also apply to its TotalF⋆ fragment. The proof of the theorem is by mutual induction on the typing derivation with the following lemmas:

Lemma 3.2 (Commutation of Subtyping).
*(a) If* $S; \Gamma \vdash t <: t_1$ *and* $S; \Gamma \vdash t : \mathsf{Type} \rightsquigarrow t'$ *then* $S; \Gamma \vdash t_1 : \mathsf{Type} \rightsquigarrow t'_1$ *and* $[|\ \Gamma\ |]_S \vdash t' <: t'_1$.
*(b) If* $S; \Gamma \vdash C <: C_1$ *and* $S; \Gamma \vdash C : \mathsf{Type} \rightsquigarrow t'$ *then* $S; \Gamma \vdash C_1 : \mathsf{Type} \rightsquigarrow t'_1$ *and* $[|\ \Gamma\ |]_S \vdash t' <: t'_1$.

*The essence of effect abstraction: Admissibility of using $F.subcomp$ for subtyping.* The reader may have noticed that the rule SC-M breaks the effect abstraction by peeking into the effect representation for checking subtyping. However, this is not necessary: we show the admissibility of checking subtyping using the $F.subcomp$ effect combinator.

The $F.subcomp$ combinator is typechecked, once-and-for-all as part of the effect definition, as follows:

$S; \cdot \vdash F.subcomp : a : \mathsf{Type} \to \overline{x : t} \to f : F.repr\ a\ \overline{e}\{t_1\} \to F.repr\ a\ \overline{e_1}$

and

$F.subcomp = \lambda\ a\ \overline{x}\ f.\ f$

where $t_1$ is a refinement formula. The intuition is that the combinator is a coercion that can be used to coerce a computation from $F\ t\ \overline{e}$ to $F\ t\ \overline{e_1}$, provided that the refinement formula is valid. The implementation of $F.subcomp$ is required to be an identity function. This is because subtyping in F⋆ is intentionally non-coercive, so that the application of subtyping does not disturb equality. To prove $F\ t\ \overline{e} <: F\ t\ \overline{e_1}$, we check that:

$S; \Gamma, f : F.repr\ t\ \overline{e} \vdash F.subcomp\ f : F.repr\ t\ \overline{e_1}$

Behind the scenes, this typechecking judgment proves the refinement formula in the type of the $F.subcomp$ combinator. The following lemma establishes the soundness of $F.subcomp$:

Lemma 3.3 (Soundness of $e_{subcomp}$). *If* $\Delta \vdash F\ t\ \overline{e} : \mathsf{Type}$, $\Delta \vdash F\ t\ \overline{e_1} : \mathsf{Type}$, *and* $\Delta, f : F.repr\ t\ \overline{e} \vdash F.subcomp\ f : F.repr\ t\ \overline{e_1}$, *then* $\Delta \vdash F\ t\ \overline{e} <: F\ t\ \overline{e_1}$.

The proof of the lemma unfolds $F.subcomp$ to prove the subtyping of the representations, after which an application of SC-M gives us the conclusion. Since it is admissible, we preserve the effect abstractions and use the $F.subcomp$ combinator to check subsumption, rather than implementing SC-M as is. Additionally, this means that when implementing type conversion in IMF⋆, we do not need to translate types all the way down to TotalF⋆—Lemma 3.3 assures us that such a translation would always succeed.

### 3.4 Encoding `PURE`, F⋆'s primitive Dijkstra monad

Prior to indexed effects, effectful computation types in F⋆ had a fixed shape $M\ a\ w$, where $w$ is an $M$-specific weakest precondition predicate transformer [Swamy et al. 2016]. The most primitive Dijkstra monad in F⋆ is for conditionally pure computations, written as `PURE (a:Type) (w:wp a)`, where `wp a` is the type of a monotonic predicate transformer, transforming an `a`-predicate into a precondition.

```
let wp a = w:((a → prop) → prop) { ∀p1 p2. (∀ x. p1 x ⟹ p2 x) ⟹ (w p1 ⟹ w p2) }
```

Rather than taking it as primitive, `PURE` can be defined as an indexed effect whose representation is `pure a w`, a form of continuation monad where the `p:(a → prop)` is a "logical continuation" in `prop`.

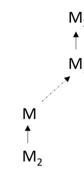
```
let pure a w = p:(a → prop) → squash (w p) → v:a{p v}
let return (x:a) : pure a (λ p → p x) = λ_ _ → x
let bind (f:pure a w1) (g:(x:a → pure b (w2 x))) : pure b (λ p → w1 (λ x → w2 x p)) =
  λp _ → let x = f (λ x → w2 x p) () in g x p () (* run f with a chosen postcondition, then run g with p *)
let subcomp (w1 w2:wp a) (_:squash (∀ p. w2 p ⟹ w1 p)) (f:pure a w1) : pure a w2 = f
let if_then_else (w1 w2:wp a) (f:pure a w1) (g:pure a w2) (b:bool) = pure a (if b then w1 else w2)
effect { PURE (a:Type) (w:wp a) with { repr = pure; return; bind; subcomp; if_then_else }}
```

`PURE a w` is the type of a pure computation whose result satisfies `p`, provided `w p` holds.

### 3.5 Implementation of indexed effects

We have implemented indexed effects in the F⋆ typechecker and all the examples presented in the paper are supported by our implementation. Below we discuss some implementation aspects.

*Coherence of lifts and effect upper bounds.* When adding a $\mathsf{lift}_{M}^{M_1}$ to the signature, our implementation computes all the new lift edges that it induces via transitive closure. For example, if $\mathsf{lift}_{M_2}^{M}$ and $\mathsf{lift}_{M_1}^{M_3}$ already exist, this new lift induces a $\mathsf{lift}_{M_2}^{M_3}$ via composition. For all such new edges, if the effects involved already have an edge between them, F⋆ ignores the new edge and emits a warning. Further, F⋆ also ensures that for all effects $M$ and $M_1$, either they cannot be composed or they have a unique least upper bound. This ensures that the final effect $M$ is unique in the rules T-Let and T-Case. Finally, F⋆ ensures that there are no cycles in the effects ordering.




*Algorithmic subtyping.* F⋆ implements a kind of bidirectional type inference algorithm, combined with constraint-based higher-order unification. By propagating programmer-provided annotations through a typing derivation, subtyping is applied only when there is an expected type from the context. Our implementation piggybacks on this infrastructure, relating computation types with the `subcomp` combinators whenever needed.

*Effect combinator for composing branches of a conditional.* While in IMF⋆ we have formalized a dependent pattern matching case, our implementation allows for specifying an optional custom effect combinator for combining branches. The shape of the combinator is as follows:
$S; \cdot \vdash F.\mathit{if\_then\_else} : a{:}\mathsf{Type} \to \overline{x{:}t} \to f{:}F.\mathit{repr}\ a\ \overline{e_{then}} \to g{:}F.\mathit{repr}\ a\ \overline{e_{else}} \to b{:}\mathit{bool} \to \mathsf{Type}$
and
$F.\mathit{if\_then\_else} = \lambda\ a\ \overline{x}\ f\ g\ b.\ F.\mathit{repr}\ a\ \overline{e_{composed}}$
F⋆ ensures the soundness of the combinator by checking that under the assumption $b$, the type of $f$ is a subtype (as per $F.\mathit{subcomp}$) of the composed type, similarly for $g$ under the corresponding assumption *not* $b$. When the combinator is omitted, F⋆ chooses a default one that forces the computation type indices for the branches to be provably equal.

*Inferring effect indices using higher-order unification.* Our implementation relies on the higher-order unifier of F⋆ to infer effect indices and arguments of the effect combinators. For example, suppose we have a computation type $F\ t_1\ \overline{e_1}$ and we want to apply the $\mathsf{lift}_{F}^{F'}$ combinator, where $\mathsf{lift}_{F}^{F'} = e$ such that:
$S; \cdot \vdash e : a : \mathsf{Type} \to \overline{x : t} \to F.\mathit{repr}\ a\ \overline{e_f} \to F'.\mathit{repr}\ a\ \overline{e}$
To apply this combinator, we create fresh unification variables for the binders $a$ and $\overline{x}$, and substitute them with the unification variables in $F.\mathit{repr}\ a\ \overline{e_f}$ and $F'.\mathit{repr}\ a\ \overline{e}$, without unfolding the $e_{repr}$. We then unify $t_1$ with the unification variable for $a$, $\overline{e_1}$ with substituted $\overline{e_f}$, and return (substituted) $F'\ a\ \overline{e}$ as the lifted computation type. This allow us to compute instantiations of the combinators without reifying $F\ t_1\ \overline{e_1}$ or reflecting the result type of lift. We follow this recipe for all the effect combinators.

In our `GST` state increment example, to bind the two computation types `GST int R` (for `read`) and `GST unit RW` (for `write`), using the `bind` combinator:

```
let bind a b t0 t1 (f:gst a t0) (g: a → gst b t1) : gst b (t0 ⊕ t1) = ...
```

we create fresh unification variables $?u_a$, $?u_b$, $?u_{t_0}$, $?u_{t_1}$ for the `a`, `b`, $t_0$, $t_1$ arguments. We then unify the indices of the `f` argument, i.e. $?u_a$ and $?u_{t_0}$, with the indices of the first computation type `int` and `R`. Similarly, for the `g` argument, $?u_b$ and $?u_{t_1}$ are unified with `unit` and `RW`. Finally, the returned computation type is `GST` $?u_b$ ($?u_{t_0} \oplus ?u_{t_1}$), which after solving for unification variables becomes `GST unit RW`.

*Support for divergence.* Our implementation also supports the existing Div effect in F⋆ for classifying divergent computations. To ensure consistency, the logical core of F⋆ is restricted to the pure fragment, separated from Div using the effect system. When defining indexed effects, the representation types may encapsulate Div computations. An indexed effect $F$ may optionally be marked divergent. When so, the semantic termination checker of F⋆ is disabled for $F$ computations. However, reification of such effects results in a Div computation to capture the fact that this computation may diverge.

## 4 DIJKSTRA MONADS AND ALGEBRAIC EFFECTS

Maillard et al. [2019] present Dijkstra monads as a monad-like structure `M` indexed by a separate monad of specifications `W`, forming types of the shape `M a w`, where `w : W a`. In this section, we present a refined instance of such a construction, a *graded* Dijkstra monad, integrated within a library of algebraic effects and handlers.

Algebraic effects and handlers are a framework for modeling effects in an extensible, composable, re-interpretable manner, with strong semantic foundations [Plotkin and Power 2003] and several new languages and libraries emerging to support them [Bauer and Pretnar 2015; Brady 2013; Leijen 2017; Lindley et al. 2017; Plotkin and Pretnar 2009]. We show that algebraic effects and handlers can be encoded generically using dependent types in F⋆, and exposed to programmers as an indexed effect supporting programming in an abstract, high-level style. Further, we show that operation labels can be conveniently tracked as an index, much like in existing effect systems for algebraic effects. Also, we reconcile them with WPs for the particular case of state, proving that functional specifications can be strengthened from the intensional information of its operations, employing a unique combination of graded and Dijkstra monads.

### 4.1 A graded monad for algebraic effects

Our starting point is a canonical free monad representation $\texttt{tree}_0$ `a` of computations with generic actions producing `a`-typed results. We include stateful operations (`Read` and `Write`) and exceptions (`Raise`), but other operations can be easily added.[6]

```
type op = | Read | Write | Raise
let op_inp o = match o with | Read → unit | Write → state | Raise → exn
let op_out o = match o with | Read → state | Write → unit | Raise → empty
type tree₀ (a : Type) = | Return : a → tree₀ a
                        | Op : op:op → i:(op_inp op) → k:(op_out op → tree₀ a) → tree₀ a
```

The type $\texttt{tree}_0$ contains all possible combinations of the operations. To limit the operations that may appear in a computation, our representation type `tree` is indexed by a set of operations that over-approximates the operations in the computation. Specifically, a computation `abides` by a set of labels `labs` if its operations are a subset of `labs`. We use a `list` for the index, but only interpret it via membership, so order and multiplicity are irrelevant. This makes `tree` a graded monad, where the monoid operation is the set union.

```
let ops = list op        let tree (a:Type) (labs:ops) = c:(tree₀ a){abides labs c}
where let rec abides (labs:ops) (c : tree₀ a) : prop = match c with
        | Return _ → ⊤| Op a i k → a ∈ labs ∧ (∀ o. abides labs (k o))
```

[6]Our implementation in the supplementary material contains an additional uninterpreted `Other : int → op`, and never relies on knowing the full set of operations.

*Packaging the refined free monad as an effect.* Elevating tree to an indexed effect is straightforward, only requiring to define the basic combinators, with most of the heavy lifting done by F$^\star$'s SMT backend. We also provide a subsumption rule to grow the labels where needed, and also reorder and deduplicate them, since they are essentially sets.

```
let return a (x:a) : tree a [] = Return x
let bind a b labs1 labs2 (c : tree a labs1) (f : a → tree b labs2) : tree b (labs1 ∪ labs2) = (* elided *)
let subcomp a labs1 labs2 (_:squash (labs1 ⊆ labs2)) (f : tree a labs1) : tree a labs2 = f
let cond (a:Type) (labs1 labs2 : ops) (f:tree a labs1) (g:tree a labs2) (p: bool) = tree a (labs1 ∪ labs2)
effect { Alg (a:Type) (labs:ops) with { repr = tree; return; bind; subcomp; if_then_else = cond }}
```

### 4.2 Operations and their handlers, with `reflect` and `reify`

To add operations to our new effect Alg, the reflect operator is useful, as seen in the generic action geneff below, which uses reflect to promote a tree to an Alg. Specific instances of operations can be defined easily using geneff, where for raise we take an extra step of matching on its (empty) result to make it polymorphic.

```
let geneff (o : op) (i : op_inp o) : Alg (op_out o) [o] = Alg.reflect (Op o i Return)
let get () : Alg state [Read] = geneff Read ()    let put (s:state) : Alg unit [Write] = geneff Write s
let raise (e:exn) : Alg α  [Raise] = match geneff Raise e with (* empty match *)
```

With this, we can already write simple programs in direct style, while the system infers refined types and elaborates programs to their tree representation.

```
exception Failure of string
let add_st x : Alg int [Read; Raise] = let s = get () in if s < 0 then raise (Failure "error") else x+s
```

When defining effect handlers, one needs access to the tree representation. For instance, the handle_tree combinator allows all the $labs_0$ operations in c to be handled by h, which in turn may leave the $labs_1$ operations to be handled, with v the continuation of c's return.

```
let handler_tree_op o b labs = op_inp o → (op_out o → tree b labs) → tree b labs
let handler_tree labs0 b labs1 = o:op{o ∈ labs0} → handler_tree_op o b labs1
let rec handle_tree (c : tree a labs0) (v : a → tree b labs1) (h : handler_tree labs0 b labs1)
  : tree b labs1 = match c with | Return x → v x
                                | Op act i k → h act i (λ o → handle_tree (k o) v h)
```

However, rather than calling handle_tree directly, which would require client code to work with tree, we provide the following interface instead, using reify in negative positions to coerce Alg to tree and reflect to move back.

```
let handler labs0 b labs1 = o:op{o ∈ labs0} → op_inp o → (op_out o → Alg b labs1) → Alg b labs1
let handle_with (f : unit → Alg a labs0) (v : a → Alg b labs1) (h : handler labs0 b labs1) : Alg b labs1
  = (* elided, essentially a wrapper of handle_tree, translating h into a handler_tree, etc. *)
```

This allows us to write handlers in a direct, applicative notation, close to what is offered by languages specifically designed for algebraic effects.

```
let defh : handler labs b labs = λo i k → k (geneff o i) (* essentially rebuilding an Op node *)
let catchE (f : unit → Alg a (Raise::labs)) : Alg (option a) labs =
  handle_with f Some (function Raise → (λ i k → None) | _ → defh)
```

### 4.3 `AlgWP`: A graded Dijkstra monad for stateful `Alg` programs

While `Alg` above bounds the operations each computation may invoke, there is no way to specify their order, or any property about the final value and state. To do so, we can bring back the idea of using a WP calculus to the algebraic setting by adding a new index to the effect. We limit ourselves to stateful operations and use WPs to specify the behavior according to the state monad: without fixing an interpretation, it is unclear what can be verified, unless equations are added.

Our new effect will refine `tree` with a stateful WP. We can define a function that takes a computation tree and computes a "default" stateful WP from it. Then, we take the representation `treewp a l wp` to be computation trees with operations in `l` whose default WP is pointwise weaker than its annotated WP (allowing underspecification). This construction is due to Maillard et al. [2019] but our setting is more general due to the additional grading, not supported in Dijkstra monads. We begin by defining a predicate transformer monad for stateful programs—`st_wp a` is the type of functions transforming postconditions on results and states to preconditions on states.

```
type post_t (a:Type) = a → state → prop
type st_wp (a:Type) = state → post_t a → prop
let read_wp : st_wp state = λs₀ p → p s₀ s₀
let write_wp : state → st_wp unit = λs _ p → p () s
let return_wp (x:a) : st_wp a = λs₀ p → p x s₀
let bind_wp (w₁ : st_wp a) (w₂ : a → st_wp b) : st_wp b = λs₀ p → w₁ s₀ (λ y s₁ → w₂ y s₁ p)
```

Next, we interpret `Read`-`Write` trees into `st_wp` via `interp_as_wp`. We refine the `tree` type by both limiting its operations and adding a refinement comparing its WP via ($\preccurlyeq$), the strengthening relation on stateful WPs, and defining `return`, `bind`, `subcomp`, `cond`, `get` and `put` for `treewp`, and promote it to the `AlgWP` effect. `AlgWP` is proven sound by interpreting it into the `PURE` effect from §3.4.

```
let rec interp_as_wp #a (t : tree a [Read; Write]) : st_wp a = match t with
  | Return x → return_wp x
  | Op Read _ k → bind_wp read_wp (λ s → interp_as_wp (k s))
  | Op Write s k → bind_wp (write_wp s) (λ () → interp_as_wp (k ()))
type rwops = l:ops{l ⊆ [Read; Write]}
let treewp (a : Type) (l:rwops) (w: st_wp a) = t:(tree a l){ w ≼ interp_as_wp t }
effect { AlgWP (a:Type) (l:rwops) (w:st_wp a) with { repr = treewp; ...}}
let soundness (t : unit → AlgWP a l wp) : s₀:state → PURE (a & state) (wp s₀) = ...
```

Using this graded Dijkstra monad, we can verify functional correctness properties, which a graded monad alone cannot capture. For instance, when the state is instantiated to a heap (mapping locations to values), we can prove that the program below correctly swaps two references, where `AlgPP` is simply a pre-/postcondition alias to `AlgWP`.

```
effect AlgPP a l p q = AlgWP a l (λ s0 post → p s0 ∧ (∀ x s1. q x s1 ⟹ post x s1))
let swap (l₁ l₂ : loc) : AlgPP unit [Write; Read] (requires λ_ → l₁ ≠l₂)
                          (ensures λh₀ _ h₁ → h₀\{l₁;l₂} == h₁\{l₁;l₂} ∧ h₁.[l₁] == h₀.[l₂] ∧ h₁.[l₂] == h₀.[l₁])
  = let r = !l₂ in l₂ := !l₁; l₁ := r
```

More interestingly, the static information in the label index can be exploited by the WP. The `quotient` function below strengthens the postcondition of a write-free `AlgWP` program into additionally ensuring that the state does not change. Operationally, `quotient` just runs `f ()`, so it can be seen as a proof that `f` does not change the state.

```
val quotient (f : unit → AlgPP a [Read] p q) : AlgPP a [Read] p (λ h₀ x h₁ → q h₀ x h₁ ∧ h₀ = h₁)
```

In summary, this case study has shown that we can develop dependently typed libraries for sophisticated effect disciplines, provide reasoning principles for them in the form of novel, hybrid indexed monads, and package it up as an effect that enables programs and their proofs to developed at a palatable, high-level of abstraction.

## 5 LAYERED INDEXED EFFECTS FOR MESSAGE FORMATTING IN TLS

Indexed effects are not just for defining new effect typing disciplines—effect layers stacked upon *existing* effects can make client programs and proofs more abstract, without any additional runtime overhead. We demonstrate this at work by stacking two effect layers over EverParse [Ramananandro et al. 2019], an existing library in F⋆ for verified low-level binary message parsing and formatting, simplifying different aspects of the programs and proofs in each layer.

*Background: EverParse and Low⋆.* EverParse is a parser generator for low-level binary message formats, built upon a verified library of monadic parsing and formatting combinators. It produces parsers and formatters verified for memory-safety (no buffer overruns, etc.) and functional correctness (the parser is an inverse of the formatter). EverParse is programmed in Low⋆, a DSL in F⋆ for C-like programming [Protzenko et al. 2017]. Low⋆'s central construct is the `Stack` effect which models programming with mutable locations on the stack and heap, with explicit memory layout and lifetimes. `Stack` is a Hoare monad with the following signature:

```
effect Stack (a:Type) (pre:mem → prop) (post:mem → a → mem → prop)
```

Programs in `Stack` may only allocate on the stack, while reading and writing both the stack and the heap, with pre- and postconditions referring to `mem`, a region-based memory encapsulating both stack and heap. Low⋆ provides fine-grained control for general-purpose low-level programming, at the expense of low-level proof obligations related to spatial and temporal memory safety, and framing–we aim to simplify these proofs for binary message formatters with domain-specific abstractions built using indexed effects.

*The problem: Existing code mired in low-level details.* Consider, for instance, formatting a struct of two 32-bit integer fields into a mutable array of bytes, a `buffer U8.t` in Low⋆ parlance.

```
type example = { left: U32.t; right: U32.t }
```

EverParse generates a lemma stating that if the output buffer contains two binary representations of integers back to back, then it contains a valid binary representation of an `example`:

```
val example_intro mem (output: buffer U8.t) (offset_from: U32.t) : Lemma
  (requires valid_from parse_u32 mem output offset_from ∧
            valid_from parse_u32 mem output (offset_to parse_u32 mem output offset_from))
  (ensures valid_from parse_example mem output offset_from)
```

To format a value of this type, one must write code like this:

```
let write_example (output: buffer U8.t) (len: U32.t) (x y: U32.t) : Stack bool
  (requires λm0 → live m0 output ∧ len == length output)                 (* memory safety *)
  (ensures λm0 success m1 → modifies output m0 m1 ∧                 (* memory safety & framing *)
    (success ⟹ valid_from parse_example m1 output 0))            (* output correctness wrt parser *)
= if len < 8 then false (* output buffer too small *)
  else let off = write_u32 output 0 x in let _ = write_u32 output off y in
      let mem = get () in example_intro mem output 0; true
```

The user needs to reason about the concrete byte offsets: they need to provide the positions where values should be written, relying on `write_u32` returning the position just past the 32-bit integer it wrote in memory. Then, they have to apply the validity lemma: satisfying its precondition involves (crucially) proving that the writing of the second field write does not overlap the first one, through Low★ memory framing. These proofs are here implicit but still incur verification cost to the SMT solver; as the complexity of the structs increases, it has a significant impact on the SMT proof automation. Moreover, the user also needs to worry about the size of the output buffer being large enough to store the two integer fields.

We describe next how using indexed effects we can reduce the programming and proving overhead to:

```
let write_example (x y : U32.t) : FWrite unit parse_empty parse_example = write_u32 x; write_u32 y
```

### 5.1 The `Write` effect

We define a `Write` effect, layered over `Stack`, to abstract the low-level byte layout, memory safety, and error handling–we will address framing later. An effectful computation `f : Write a pbefore pafter` returns a value of type `a` while working on a hidden underlying mutable buffer. Each such computation requires upon being called that the buffer contains binary data valid according to the `pbefore` parser specification, and ensures that, if successful, it contains binary data valid according to `pafter` on completion. Thus, `Write` is a simple parser-indexed parameterized state and error monad, that hides the mundane memory safety, binary layout, and error propagation details from its clients. Returning to `example`, we can define:

```
(* A leaf writer for writing an integer *)
val write_u32 : U32.t → Write unit parse_empty parse_u32
(* A generic higher–order framing operator, to be able to write two pieces of data in a row *)
val frame (#a: Type) (#pframe #pafter : parser) (f: unit → Write a parse_empty pafter)
  : Write a pframe (parse_pair pframe pafter)
(* A lifting of the example_intro_mem lemma, with all binary layout details hidden, computationally a no–op. *)
val write_example_correct : unit → Write unit (parse_pair parse_u32 parse_u32) parse_example
```

This last lemma states that, if the output buffer contains valid data for parsing a pair of two integers, then calling this function will turn that data into valid data for parsing an `example` struct value. With these components, the user can now write their formatting code more succinctly, as shown below. The output buffer, offsets, and error propagation are hidden in the effect and so, the user no longer needs to explicitly reason about them. Furthermore, the code becomes much more self-explanatory.

```
let write_example (x y : U32.t) : Write unit parse_empty parse_example =
  write_u32 x; frame (λ _ → write_u32 y); write_example_correct ()
```

*Defining* `Write`*: A peek beneath the covers.* We represent `Write` using a dependent pair of indexed monads (`p.datatype` is the type of the values parsed by `p`):

```
type repr (a : Type) pbefore pafter = (spec : repr_spec t pbefore.datatype pafter.datatype
                                        & repr_impl t pbefore pafter spec)
```

The first field, `spec`, is a specificational parameterized monad evolving an abstract state from `pbefore.datatype` to `pafter.datatype`. The second field is the Low★ implementation, indexed by a pair of parsers and the `spec`. As such, `repr_impl` is a parameterized-monad-indexed monad, or a form of parameterized Dijkstra monad, a novel construction, as far as we are aware.

*Compiling* `Write` *computations to C.* To compile a `Write` computation to C, or call it from other Low★ code, we simply reify it and project its Low★ implementation:

```
let reify_spec (f: unit → Write a pbefore pafter)
  : repr_spec a pbefore.datatype pafter.datatype = fst (reify (f ()))
(* Extract the Low* code of a computation to compile it to C *)
let reify_impl (f: unit → Write a pbefore pafter)
  : repr_impl a pbefore pafter (reify_spec f)) = snd (reify (f ()))
```

*Using effect subcomp for automatic parser rewriting.* We can embed parser rewriting rules in the subcomp combinator for `Write`, as follows:

```
val subcomp a (p1 : parser) (p2 p2' : parser{valid_rewrite p2 p2'}) (_ : repr a p1 p2) : repr a p1 p2'
```

where `valid_rewrite p2 p2'` is a relation on parser specifications stating that any binary data valid for `p2` is also valid for `p2'`. Since the `valid_rewrite` goals can automatically be solved via SMT, this allows us to rewrite `example` as simply: `(write_u32 x; frame (λ_ → write_u32 y))`. The remaining overhead is framing; we eliminate it with another indexed effect layered on top of `Write`.

### 5.2 Automated framing with `FWrite`

Following the frame inference methodology proposed by Fromherz et al. [2021] in the context of a concurrent separation logic, we define a new effect `FWrite` that automatically adds frames to the computations when sequentially composing them.

```
type frepr (a:Type) pbefore pafter = unit → Write a pbefore pafter
val fbind (a b: Type) (p1 p1' p2 p2': parser)
  (frame_f: parser) (frame_g: parser)
  (_ : squash (valid_rewrite (parse_pair frame_f p2) (parse_pair frame_g p1')))
  (f: frepr a p1 p2) (g: a → frepr b p1' p2')
  : frepr b (parse_pair frame_f p1) (parse_pair frame_g p2')
effect { FWrite (t: Type) (pbefore pafter: parser) with { repr = frepr; bind = fbind; ... } }
```

The `fbind` combinator inserts frames `frame_f` and `frame_g` to the two computations `f` and `g`, and adds a squashed goal to the VC ensuring that the framed postcondition of `f`, `parse_pair frame_f p2`, can be rewritten into the framed precondition of `g`, `parse_pair frame_g p1'`. Under the hood, `FWrite` computations are thunked `Write` computations; implementing `fbind` thus consists of composing calls to `f` and `g` encapsulated by the `frame` combinator of the `Write` effect.

To automatically infer `frame_f` and `frame_g`, and discharge the framing related VCs, similar to Fromherz et al. [2021], we gather all the framing goals and implicits and discharge them using a (partial) decision procedure that we implement as an $F^\star$ tactic.

With `FWrite`, we can now write `example` in, arguably, the most natural way:

```
let write_example (x y : U32.t) : FWrite unit parse_empty parse_example = write_u32 x; write_u32 y
```

By successively layering several effects, we thus retrieve a proof style akin to proofs by refinement, but for effectful computations. We abstract away reasoning about error handling, low-level byte layout, and framing, through different indexed effects, finally providing a programmer with a high-level interface closer to an idealized functional program to use verified low-level serializers.

The `FWrite` effect scales to more than just writing a record of two integers. We show how to write a variable-sized list of 32-bit integers, the list being prefixed by a header recording its size in bytes. If `p` is a parser for the elements of the list, then `parse_vllist p min max` is a parser that first reads a header consisting of an integer value that will be the total storage size of the list elements in bytes, then checks that it is between `min` and `max`, then parses the list of elements using `p` for each header. The `min` and `max` bounds are constants mandated by the data format and independent of the size of the actual output buffer. The following code writes a list of two integers, following the data format specified by parse_vllist:

```
let write_int_list (max_list_size: U32.t)
  : FWrite unit parse_empty (parse_vllist parse_u32 0 max_list_size) =
  write_vllist_nil parse_u32 max_list_size;
  write_u32 18ul; extend_vllist_snoc ();
  write_u32 42ul; extend_vllist_snoc ()
```

The value of max_list_size constrains the size of the size header, but thanks to the abstraction provided by Write (and hence FWrite), the user does not need to know about that actual size. The code relies on two combinators:

```
val write_vllist_nil (p: parser) (max: U32.t) : FWrite unit parse_empty (parse_vllist p 0 max)
val extend_vllist_snoc (#p: parser) (#min #max: U32.t) ()
  : FWrite unit (parse_pair (parse_vllist p min max) p) (parse_vllist p min max)
```

write_vllist_nil starts writing an empty list by writing 0 as its size header. extend_vllist_snoc assumes that the output buffer contains some variable-sized list immediately followed by an additional element and "appends" the element into the list by just updating the size header of the list; thus, the new element is not copied into the list, since it is already there at the right place. extend_vllist_snoc also dynamically checks whether the size of the resulting list is still within the bounds expected by the parser, returning an error if not.

The data format specified by parse_vllist p min max and implemented by those FWrite combinators corresponds to the format of variable-sized lists prefixed by their byte size as mandated by the TLS 1.3 RFC [Rescorla 2018].

### 5.3 Application: TLS 1.3 handshake extensions

We have used the Write effect to generate the list of extensions of a TLS 1.3 [Rescorla 2018] *ClientHello* handshake message, that a client sends to a server to specify which cipher suites and other protocol extensions it supports. This is the most complex part of the handshake message format, involving much more than just pairs: it involves variable-sized data and lists prefixed by their size in bytes (as in the write_int_list example above), as well as tagged unions where the parser of the payload depends on the value of the tag. Our Write effect based implementation of ClientHello messages compiles to C and executes; we are currently rewriting it with FWrite to take advantage of automated framing, and integrating it into a low-level rewriting of an implementation of TLS in F⋆ [Bhargavan et al. 2013].

A more powerful version of the Write and FWrite effects with support for Hoare-style pre- and postconditions to prove functional correctness properties on the actual values written to the output buffer, as well as error postconditions, in addition to correctness with respect to the data format, is underway. With such an enhanced version, we plan to leverage pre- and postconditions to avoid dynamic checks on writing variable-size list items.

## 6 EXISTING APPLICATIONS OF INDEXED EFFECTS

Indexed effects have been available in recent releases of F⋆ and have been used in Steel [Fromherz et al. 2021] and DY⋆ [Bhargavan et al. 2021], two independent developments. These uses validate our design and support our claim that indexed effects help structure effectful programs and proofs at scale. We briefly summarize their work, while refering the reader to the Steel and DY⋆ papers for more details.

## 6.1 Steel

Steel is a language for developing and proving concurrent, dependently-typed F⋆ programs. Steel's program logic is based on a shallow embedding of concurrent separation logic in F⋆, while also enabling Hoare-style reasoning using a variant of heap predicates, called *selector predicates*.

To enable smooth interoperation between separation logic and Hoare logic, Steel encodes pre- and postconditions as effect indices; we show a simplified version of the Steel effect representation:

```
let selpre fp = rheap fp → prop          let selpost fp a fp' = rheap fp → a → rheap fp' → prop
val steel_repr (a:Type) (fp:slprop) (fp':a → slprop) (req:selpre fp) (ens:selpost fp a fp') : Type
```

Separation logic specifications rely on assertions of type `slprop`, and are encoded through a precondition `fp`, and a return value dependent postcondition `fp'`. Similarly to the Hoare monad from §2.3, selector specifications consist of a precondition `req` and a 2-state postcondition `ens`. Note that `req` and `ens` operate on states (i.e., heaps) that are parameterized by the separation logic specifications: `rheap fp` is a restricted heap corresponding exactly to the predicate `fp`. This is another instance of a hybrid, dependent indexing structure, where the `fp` indices constrain the selector predicates.

$$\begin{tikzcd} \text{Steel} \\ \text{SteelAtomic} \arrow[u, dotted] \\ \text{SteelGhost} \arrow[u, dotted] \end{tikzcd}$$

To reason about concurrent programs, Steel also models atomic computations, as well as a notion of ghost state, which can be manipulated through ghost, computationally irrelevant computations. Ghost and atomic computations are separated from generic Steel functions, and are thus modeled as their own effects, `SteelGhost` and `SteelAtomic`, with two additional effect indices to encode verification conditions related to atomicity. Nevertheless, a ghost computation can always be seen as atomic, while an atomic computation is but a special case of a generic Steel computation. Steel captures this hierarchy through lifts between its different effects, which are automatically inserted by our framework when needed.

In Steel, indexed effects thus provide a foundation to structure reasoning, enabling, for instance, a separation of verification conditions related to atomicity and separation logic. Leveraging this structure, Steel automates framing reasoning, using a methodology similar in spirit to the one presented in §5.2, albeit applied to a full-fledged, impredicative, concurrent separation logic, which simplified the development of a wide variety of verified libraries, ranging from self-balancing trees and concurrent queues to 2-party session types.

## 6.2 DY⋆

DY⋆ is an F⋆-based framework for symbolic verification of security protocols and has been used for the first symbolic analysis of the Signal protocol, the messaging protocol used in WhatsApp, while accounting for an unbounded number of ratcheting rounds.

Protocols sessions in DY⋆ are modeled as partial, stateful F⋆ functions that may raise exceptions and the underlying state is a global, monotonic trace that tracks the interleaved execution of sessions. This is represented as an indexed effect for a state and exception Dijkstra monad, called `Crypto`. All the security protocols in DY⋆, including Signal, are written in the `Crypto` effect, and verified against the trace-based properties expressed as specifications in the Dijkstra monad.

```
type wp (a:Type) = (option a → trace → prop) → trace → prop
(* The monotonicity property of the trace is internalized in the repr via extends *)
type crypto_repr (a:Type) (w:wp a) =
  s0:trace → PURE (option a & trace) (λ p → w (λ x s1 → s1 `extends` s0 ⟹ p (x, s1)) s0)
```

Without indexed effects, defining such an effect in F⋆ would not be possible, and using an axiomatized effect for verifying security properties is clearly suboptimal. Though one could derive an effect for state and exceptions using the Dijkstra Monads for Free methodology [Ahman et al. 2017], it does not allow internalizing trace monotonicity, as they have done here.

## 7 RELATED WORK & CONCLUSIONS

We have discussed several strands of related work throughout the paper. We focus here on relating our work to two main themes not discussed in detail elsewhere.

*Unifying frameworks for effectful programming and semantics.* Given the variety of monad-like frameworks for effects, a unifying theory that accounts for all the variants is a subject of some interest. Filinski [1999] presents a framework for specifying and implementing layered monads, focusing on implementing them uniformly with delimited continuations and state, though does not consider indexed monads. Tate's (2013) productors and, equivalently, Hicks et al.'s (2014) polymonads are attempts at subsuming frameworks, Tate focusing more on the semantics while Hicks et al. consider programmability, though neither handle dependently typed programs. Bracker and Nilsson [2015] provide a Haskell plugin for polymonad programming—our implementation also provides support for polymonads, where not only the indices but also the effect label can vary among the computation arguments to `bind`, a feature used in Steel [Fromherz et al. 2021]. Orchard et al. [2020] propose to unify graded and parameterized monads by moving to category-indexed monads, studying them from a semantic perspective only, while also working in a simply typed setting. Orchard and Petricek [2014] also propose a library to encode effect systems with graded monads in Haskell.

*Programming and proving with algebraic effects.* Our library for algebraic effects is perhaps related most closely to Brady's (2013) Effects DSL in the dependently typed language Idris. The main points of difference likely stem from what is considered idiomatic in Idris versus F⋆. For instance, the core construct in the Idris DSL is a type indexed by a list of effects (similar to our `tree a l`)—whereas in Idris the indexing is intrinsic, our trees are indexed extrinsically with a refinement type, enabling a natural notion of subsumption on indices based on SMT-automated effect inclusion. Idris' core effects language is actually a parameterized monad—our supplement and full version of the paper show a similarly parameterized version of our tree type. By packaging our trees into an effect, we benefit from automatic elaboration, avoiding the need for monadic syntax, idiom brackets and the like, with implicit subsumption handled by SMT. Further, unlike Brady, we provide a way to interpret Read-Write trees into a Dijkstra monad, enabling functional correctness proofs. While we have only taken initial steps in this direction, we appear to be the first to actually verify stateful programs in this style. Maillard et al. [2019] propose a tentative semantics to interpret algebraic effect handlers with Dijkstra monads, and use their approach to extrinsically verify the totality of a Fibonacci program with general recursion. Our work builds on theirs, requires fixing the interpretation of the operations, but yields a methodology to do proofs of stateful programs. Besides, with indexed effects, we get to choose whether to work with Dijkstra monads or not—in contrast, Maillard et al.'s framework cannot support the list-of-effects indexed graded monad. Algebraic effects have also been embedded in Haskell in several styles, notably by Kiselyov and Ishii's (2015) "freer" monads, relying on encodings of dependent types in Haskell's type system to also index by a list of effect labels, while focusing also on efficient execution, a topic we have not yet addressed for our `Alg` effect.

*Conclusions.* Embracing the diversity of indexed monad-like constructions, and aiming to benefit from them when programming with effects in a dependently typed language, we have designed and implemented *indexed effects* as a feature of F⋆. In doing so, we have simplified F⋆'s core logic, while also enabling new abstractions for programming and proving. Being available in F⋆ for the past year, we have already seen indexed effects deployed by users in various settings, giving us confidence that our work scales to large developments. By lowering the bar to programming with indexed monads, we hope to encourage the development of new indexed constructions and new patterns of proof for effectful software.

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